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#### A NOTE ON sg\* CONTINUOUS MAPPINGS IN SOFT TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce sg\*closed set in a Soft topological space and to study some of its properties. Then sg\* continuous mapping and irresolute mapping areintroduced and some of its properties are studied. The concept sg\* open, sg\* closed mappings and sg\*homeomorphism are introduced and their properties are studied.

Key-Words: sg\* continuous mapping, irresolute mapping, sg\* homeomorphism

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### **1. INTRODUCTION**

The theory of soft sets gives a vital mathematical tool for handling uncertainties and vague concepts. In the year 1999, Molodtsov[1] initiated the study of soft sets. Soft set theory has been applied in several directions. Following this Maji, Biswas, and Roy[7,8] discussed soft set theoretical operations and gave an application of soft set theory to a decision making problem. Recently Muhammad Shabir and Munazza Naz introduced the notion of soft topology[10] and established that every soft topology induces a collection of topologies called the parametrized family of topologies induced by the soft topology. Several mathematicians published papers on applications of soft sets and soft topology [1,2,6,11,12,1]. Soft sets and soft topology have applications to data mining, image processing, decision making problems, spatial modeling and neural patterns[3,4,5,7]. In this paper, the concept sg\*closed set is introduced in soft topological space and the concept of sg\* continuous mapping and sg\* irresolute mapping are introduced and some of their properties are studied. Further the concept sg\* open , sg\* closed mappings and sg\*homeomorphism are introduced and some of their basic soft topological properties are investigated. Finally the concept of slightly sg\* continuous mapping is introduced and studied some of its basic concepts.

### **2. PRELIMINARIES**

**2.1 Definition** A soft set (A, E) is called sg\* closed in a soft topological space  $(X, \tilde{r}, E)$  of  $cl(A, E) \subseteq (U, E)$  whenever  $(A, E) \subseteq (U, E)$  and (U, E) is soft g open in  $\tilde{X}$ .

2.2.1 Let 
$$X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$$
 and  
 $\tilde{r} = \{\tilde{\phi}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E), (A_7, E)\}$  where  
 $(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\}), (A_2, E) = \{(b_1, \{a_2\}), (b_2, X)\}$   
 $(A_3, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_2, a_3\})\}, (A_4, E) = \{(b_1, \{a_1, a_3\}), (b_2, X)\},$   
 $(A_5, E) = \{(b_1, \phi) \{b_2, \{a_1\}), (A_6, E) = \{(b_1, \phi) \{b_2, \{a_2, a_3\})\}$  and  
 $(A_7, E) = \{(b_1, \phi), (b_2, X)\}.$ 

Clearly  $(A, E) = \{(b_1, \{a_1, a_3\})(b_2, \{a_3\})\}$  is sg\* closed in  $(X, \tilde{r} E)$ .

since for (A,E) there exists a soft g open set  $(U,E) = \{(b_1,\{a_1,a_3\}, (b_2,\{a_2,a_3\})\}$  such that  $cl(A,E) \cong (U,E)$ .

# 2.1 Theorem

Every soft closed set is sg\* closed in a soft topological space  $(X, \tilde{r} E)$ .

# **3. sg\* CONTINUOUS MAPPINGS**

## 3.1 Definition

A soft mapping  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is called sg<sup>\*</sup> continuous if  $\mathbf{f}^1(U, E)$  is sg<sup>\*</sup> closed in  $(\mathbf{X}, \tilde{r}, E)$  for every soft closed set (U, E) of  $(\mathbf{X}, \tilde{\omega}, E)$ .

# 3.2. Theorem

Let  $\mathbf{f}: \widetilde{\mathbf{X}} \to \widetilde{Y}$  be a soft mapping from soft topological space  $(\mathbf{X}, \widetilde{r}, E)$  into a soft topological space  $(\mathbf{X}, \widetilde{r}, E)$ . Then the following statements are equivalent.

- i)  $f: \tilde{X} \to \tilde{Y}$  is sg<sup>\*</sup> continuous.
- ii) The inverse image of each soft open set in  $\tilde{Y}$  is sg<sup>\*</sup> open in  $\tilde{Y}$ .
- iii) For each soft subset  $(A, E) \in (Y, \tilde{\omega}, E) sg^* cl(\mathbf{f}^{-1}(A, E)) \subseteq \mathbf{f}^{-1} cl(A, E))$ .

iv) For each soft subset  $(B, E) \in (X, \tilde{r}, E) \mathbf{f}(sg^*cl(B, E)) \subseteq cl(\mathbf{f}(B, E))$ .

**Proof** (i)  $\rightarrow$  (ii) follows from 3.1 Definition.

(i)→(iii)

Let (A, E) be a soft subset of  $(Y, \tilde{\omega}, E)$ . By 3.2.1 Definition  $\mathbf{f}^{-1} cl(A, E)$  is a sg\* closed set containing  $\mathbf{f}^{-1}(A, E)$  and  $sg^*cl(\mathbf{f}^{-1}(A, E)) \cong \mathbf{f}^{-1}cl(A, E))$ .

(iii)  $\rightarrow$ (iv)

Let  $(B, E) \in (Y, \tilde{r}, E)$ , then  $\mathbf{f}(B, E) \in (Y, \tilde{\omega}, E)$  Hence from (iii)  $sg^*cl(\mathbf{f}^{-1}(\mathbf{f}(B, E)) \subseteq \mathbf{f}^{-1}(cl(A, E))$ . Therefore  $\mathbf{f}(sg^*cl(B, E)) \subseteq clf(B, E)$ .

$$(iv) \rightarrow (i)$$

Let (U,E) be a soft closed set in  $\tilde{Y}$ . Then by (iv)

 $\mathbf{f}(sg^*cl(\mathbf{f}^{-1}(U,E))) \cong cl(\mathbf{f}(\mathbf{f}^{-1}(U,E)) \quad . \quad \text{Hence} \quad sg^*cl(\mathbf{f}^{-1}(U,E) \cong \mathbf{f}^{-1}(U,E).$ Therefore  $\mathbf{f}^{-1}(U,E)$  is a sg\* closed set in  $\tilde{X}$ .

## 3.3 Theorem

Let  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  be a soft continuous mapping from  $\tilde{\mathbf{X}}$  into  $\tilde{Y}$ . Then it is sg\* continuous.

# Proof

(i)  $\rightarrow$  (ii) follows from 3.1 Definition.

(i)→(iii)

Let (A, E) be a soft subset of  $(Y, \tilde{\omega}, E)$ . By 3.1 Definition  $\mathbf{f}^{-1}(cl(A, E))$  is a sg\* closed set containing  $\mathbf{f}^{-1}(A, E)$  and  $sg^*cl(\mathbf{f}^{-1}(A, E)) \cong \mathbf{f}^{-,\{1\}}(cl(A, E))$ .

(iii)  $\rightarrow$ (iv)

Let  $(B, E) \cong (X, \tilde{r}, E)$ . Then  $\mathbf{f}(B, E) \in (Y, \tilde{\omega}, E)$ . Hence from (iii)  $sg^*cl(\mathbf{f}^{-1}(\mathbf{f}(B, E)))$  $\cong \mathbf{f}^{-1}(clf(B, E))$ . Therefore  $\mathbf{f}(sg^*cl(B, E)) \cong clf(B, E)$ .

 $(iv) \rightarrow (i)$ 

Let (U,E) be a soft closed set in  $\tilde{Y}$ . Then by (iv)

$$\mathbf{f}(sg^*cl(\mathbf{f}^{-1}(U,E))) \cong cl(\mathbf{f}(\mathbf{f}^{-1}(U,E)). \text{ Hence } sg^*cl(\mathbf{f}^{-1}(U,E)) \cong \mathbf{f}(U,E).$$

Therefore  $f^{-1}(U, E)$  is a sg\* closed set in  $\tilde{X}$ .

### 3.4 Theorem

Let  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  be a soft continuous mapping from  $\tilde{\mathbf{X}}$  into  $\tilde{Y}$ . Then it is sg\* continuous.

# Proof

Let (A, E) be any soft closed set in  $\tilde{Y}$ . Then  $f^{-1}(A, E)$  is soft closed in  $\tilde{X}$ . Therefore by 2.1 Theorem,  $f^{-1}(A, E)$  is sg\* closed in  $\tilde{X}$ .

#### 3.5 Example

The following example shows that the converse of the above 3.2.2 Theorem need not be true.

Let  $X = \{a_1, a_2, a_3\}, Y = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$  and

 $\widetilde{r_1} = \{\widetilde{\emptyset}, \widetilde{X}, (B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E)\}$ 

 $\tilde{r_1} = \{ \widetilde{\emptyset}, \widetilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E) \}$  be two soft topological spaces over X and Y respectively. Then  $(B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E)$  are soft sets over X and  $(A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E)$  are soft sets over Y defined as follows:

$$(A_{1}, E) = \{(b_{1}, \{a_{2}, a_{3}\}), (b_{2}, \{a_{1}, a_{3}\})\}, \qquad (A_{2}, E) = \{(b_{1}, \{a_{3}\}), (b_{2}, \{a_{1}\})\}, \\ (A_{3}, E) = \{(b_{1}, \{a_{2}\}), (b_{2}, \{a_{3}\})\}, \qquad (A_{4}, E) = \{(b_{1}, \{a_{3}\}), (b_{2}, \emptyset)\}, \\ (A_{5}, E) = \{(b_{1}, X), (b_{2}, \{a_{1}, a_{3}\})\}, \qquad (A_{6}, E) = \{(b_{1}, \{a_{2}, a_{3}\}), (b_{2}, \{a_{3}\})\}, \\ (B_{1}, E) = \{(b_{1}, \{a_{2}\}), (b_{2}, \{a_{1}\})\}, \qquad (B_{2}, E) = \{(b_{1}, \{a_{3}\}), (b_{2}, \{a_{1}, a_{3}\})\}, \\ (B_{3}, E) = \{(b_{1}, \{a_{2}, a_{3}\}), (b_{2}, \{a_{1}, a_{2}\})\}, \qquad (B_{4}, E) = \{(b_{1}, X), (b_{2}, \{a_{1}, a_{2}\})\}, \\ \text{and} \quad (B_{5}, E) = \{(b_{1}, \emptyset), (b_{2}, \{a_{1}\})\}.$$

Let  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  be a soft mapping defined by  $\mathbf{f}(a_1) = a_1$ ,  $\mathbf{f}(a_2) = a_3$ , and  $\mathbf{f}(a_3) = a_2$ . Then  $\mathbf{f}$  is sg\* continuous map but not soft continuous. Since  $f^{-1}(A_1, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_2, \})\},\$ 

$$f^{-1}(A_2, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}, \qquad f^{-1}(A_3, E) = \{(b_1, \{a_3\}), (b_2, \{a_2\})\},\$$

$$f^{-1}(A_4, E) = \{(b_1, \{a_2\}), (b_2, \emptyset)\}, \qquad f^{-1}(A_5, E) = \{(b_1, X), (b_2, \{a_1, a_2\})\}, \\ f^{-1}(A_6, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_2\}) \text{ are sg* open sets in } \tilde{r_1} \text{ but} \\ f^{-1}(A_3, E), f^{-1}(A_4, E), f^{-1}(A_5, E), f^{-1}(A_6, E) \text{ are not soft open sets in } \tilde{r_1}.$$

#### 3.6 Theorem

If  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is a sg\* continuous mapping from  $\tilde{\mathbf{X}}$  into  $\tilde{Y}$  then  $\mathbf{f}$  is soft g continuous.

Proof Let (A, E) be any soft closed set in  $\tilde{Y}$ . Then  $\mathbf{f}^{-1}(A, E)$  is sg\* closed in  $\tilde{X}$ . Therefore by 2.1 Theorem  $\mathbf{f}^{-1}(A, E)$  is soft g closed in  $\tilde{X}$ .

#### 3.7 Definition

A soft mapping  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  called sg\* irresolute if  $\mathbf{f}^{-1}(U, E)$  is sg\* closed in  $\tilde{\mathbf{X}}$  for every sg\* closed set of  $(Y, \tilde{\omega}, E)$ .

# 3.8 Remark

A soft mapping  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is sg\* irresolute if and only if the inverse image of every sg\* open set in  $(Y, \tilde{\omega}, E)$  is sg\* open in  $\tilde{\mathbf{X}}$ .

**3.9 Theorem** If  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  and  $h: \tilde{Y} \to \tilde{Z}$  are any two soft mappings then

- i)  $h \circ g$  is sg\* continuous if h is soft continuous and f is sg\* continuous.
- ii)  $h \circ g$  is sg\* continuous if h is sg\* continuous and g is sg\* irresolute.
- iii)  $h \circ g$  is sg\* irresolute if both g and h are sg\* irresolute.

### Proof

(i) Let (U,E) be a soft closed set in  $\tilde{Z}$ . Then  $h^{-1}(U,E)$  is soft closed in  $\tilde{Y}$  and  $g^{-1}(h^{-1}(U,E)) = h^{\circ} g(U,E)$  is sg\* closed in  $\tilde{X}$ .

(ii) Let (U,E) be a soft closed set in  $\tilde{Z}$ . Then  $h^{-1}(U,E)$  is sg<sup>\*</sup> closed in  $\tilde{Y}$  and  $g^{-1}(h^{-1}(U,E)) = h^{\circ} g(U,E)$  is sg<sup>\*</sup> closed in  $\tilde{X}$ .

(iii) Let (U,E) be a sg<sup>\*</sup> closed set in  $\tilde{Z}$ . Then  $h^{-1}(U,E)$  is sg<sup>\*</sup> closed in  $\tilde{Y}$  and  $g^{-1}(h^{-1}(U,E)) = h^{\circ} g(U,E)$  is sg<sup>\*</sup> closed in  $\tilde{X}$ .

### 3.10 Theorem

A soft mapping  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is sg<sup>\*</sup> irresolute if and only if for every soft subset (U,E) of  $\tilde{\mathbf{X}}, g(sg^* cl(U,E)) \cong sg^* cl(g(U,E))$ .

**Proof** Let g be a sg\* irresolute mapping and (U,E) be a soft subset in  $\tilde{X}$ . Then  $sg^* cl(g(U,E))$  is sg\* closed set in  $\tilde{Y}$ . Hence  $g^{-1}(sg^* cl(g(U,E)))$  is sg\* closed in  $\tilde{X}$  and  $(U,E) \subseteq g^{-1}(g(U,E)) \subseteq g^{-1}(sg^* cl(g(U,E)))$ .

Therefore

 $sg^* cl(U, E) \subseteq g^{-1}(sg^* cl(g(U, E)))$ , hence  $g(sg^* cl(U, E)) \subseteq g^{-1}(sg^* cl(g(U, E)))$ .

Conversely, suppose that (U,E) is sg<sup>\*</sup> closed in  $\tilde{Y}$ .

Therefore

 $g(sg^* cl(g^{-1}(U,E))) \cong (sg^* cl(g(g^{-1}(U,E))) = sg^* cl(U,E) = (U,E).$  Hence  $sg^* cl(g^{-1}(U,E)) \cong g^{-1}(U,E).$ 

# 4. sg\* HOMEOMORPHISMS

# 4.1 Definition

A soft mapping  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is called sg\* open if g(U, E) of each soft open set (U, E) in

 $(X, \tilde{r}, E)$  is sg\* open in  $(Y, \tilde{\omega}, E)$ .

# 4.2 Definition

A soft mapping  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is called sg<sup>\*</sup> closed if g(U, E) of each soft closed set (U, E) in  $(\mathbf{X}, \tilde{r}, E)$  is sg<sup>\*</sup> closed in  $(Y, \tilde{\omega}, E)$ .

## 4.3 Theorem

Let the soft mappings  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  and  $g: \tilde{Y} \to \tilde{Z}$  be bijective. If  $g \circ \mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Z}$  is soft continuous and  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is soft continuous and  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is soft continuous. Z is sg\* continuous.

### Proof

Let (U,E) be the soft closed set in  $\tilde{Z}$ . Since  $g \circ f: \tilde{X} \to \tilde{Z}$  is soft continuous, then  $f^{-1}(g^{-1}(U,E)) = (g \circ f)^{-1}(U,E)$  is soft closed set in  $\tilde{X}$ . Since  $f: \tilde{X} \to \tilde{Y}$  is sg<sup>\*</sup> closed, then  $f(f^{-1}(g^{-1}(U,E))) = g^{-1}(U,E)$  is sg<sup>\*</sup> closed set in  $\tilde{Y}$ .

#### 4.5 Theorem

A soft mapping  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is a sg\* open iff if  $\mathbf{f}(ikt(B,U)) \cong sg^*ikt(\mathbf{f}(B,E))$  for every soft subset (B,E) of  $\tilde{\mathbf{X}}$ .

#### Proof

Let  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  be sg\* open and (B, E) be a soft subset of  $\tilde{\mathbf{X}}$ , then ikt(B, U) is a soft open set in  $\tilde{\mathbf{X}}$ . Hence  $\mathbf{f}(ikt(B, E)) = sg^*ikt (\mathbf{f}(ikt(B, E)))$ .

Conversely, Let (G,E) be a soft open set in  $\tilde{X}$ .  $\mathbf{f}(G,E) = \mathbf{f}(ikt(G,E)) \cong sg^*ikt (\mathbf{f}(G,E))$ , which implies  $\mathbf{f}(G,E) \cong sg^*ikt (\mathbf{f}(G,E))$ . Hence  $\mathbf{f}(G,E)$  is a sg\* open in  $\tilde{Y}$ .

### 4.6 Definition

If a soft mapping  $\mathbf{f}: \tilde{\mathbf{X}} \to \tilde{Y}$  is sg\* continuous bijective and  $\mathbf{f}^{-1}$  is sg\* continuous then *f* is said to be sg\* homeomorphism from  $(\mathbf{X}, \tilde{r}, E)$  in to  $(Y, \tilde{\omega}, E)$ .

## 4.7 Theorem

Let  $f: \tilde{X} \to \tilde{Y}$  be the soft bijective mapping. Then the following statements are equivalent: . Since f is sg\* open map,

- i)  $f^{-1}: \tilde{Y} \to \tilde{X}$  is sg\* continuous.
- ii) f is sg\* open.
- iii) f is sg\* closed.

#### Proof

(i)  $\rightarrow$  (ii) Let (U,E) be any soft open set in  $\tilde{X}$ . Since  $f^{-1}: \tilde{Y} \rightarrow \tilde{X}$  is sg<sup>\*</sup> continuous, therefore  $(f^{-1})^{-1}(U,E) = f(U,E)$  is sg<sup>\*</sup> open in  $\tilde{Y}$ .

(ii)  $\rightarrow$  (iii) Let (B,E) be any soft closed set in  $\tilde{X}$ , then  $\tilde{X} - (B,E)$  is soft open set in  $\tilde{X}$ . Since f is sg\* open map,  $f(\tilde{X} - (B,E))$  is sg\* open in  $\tilde{Y}$ . But  $f(\tilde{X} - (B,E)) = \tilde{Y} - f(B,E)$ , implies f(B,E) is sg\* closed in  $\tilde{Y}$ .

(iii)  $\rightarrow$  (i) Let (B,E) be any soft closed set in  $\tilde{X}$ . Then  $(f^{-1})^{-1}(U,E) = f(U,E)$  is sg\* closed in  $\tilde{Y}$ . Therefore  $f^{-1}: \tilde{Y} \rightarrow \tilde{X}$  is sg\* continuous.

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